# 16.482 / 16.561: Computer Architecture and Design 

## Summer 2015

Homework \#4 Solution

1. Branch history tables (50 points) Say you are executing a program that contains the following high-level code snippet:
```
A[8] = {7, 4, 3, 2, 5, 1, 6, 10};
for (i = 0; i < 8; i++) {
    if (A[i] > 4) { <fall-through code> }
    else { <branch taken code> }
}
```

When compiled, this code contains two branches, as shown below. The BNE is part of the if statement above-if the condition is true, the branch is not taken; if the condition is false, the branch is taken. The BEQ controls the end of the loop.

| $\frac{\text { Address }}{\text { Decimal }}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| 16 | $\frac{\text { Hex }}{0 \times 10}$ | loop | $\ldots$ |
| 44 |  | $0 \times 2 C$ | $\ldots$ |
|  |  | $\ldots$ |  |
| 120 | $0 \times 78$ |  | BEQ R R $4, ~ R 0, ~ R 8, ~ l o o p ~$ |

Your processor contains an eight-entry, 2-bit branch history table; its state when the processor reaches this code is as follows:

| Entry \# | $\frac{\text { Value }}{10}$ |
| :--- | :--- |
| 0 | 10 |
| 1 | 11 |
| 2 | 01 |
| 3 | 00 |
| 4 | 01 |
| 5 | 00 |
| 6 | 11 |
| 7 | 10 |

Determine the overall misprediction rate of the branch predictor for this code (in other words, what fraction of the predictions are incorrect).

Solution: The first order of business is to determine which entry of the BHT each branch accesses. Remember, we want the bits that change most often, which implies low-order address bits. We do, however, have to account for the fact that all MIPS instructions are 32 bits, so the lowest two address bits do not change. Since we have a table with $8=2^{3}$ entries, we need 3 bits to index it:

BNE: address $44=0 \times 2 \mathrm{C}=00101100 \rightarrow$ entry 3
BEQ: address $120=0 \times 78=0111 \mathbf{1 0 0 0} \rightarrow$ entry 6

We now need to determine the actual outcomes of the branches in the program. Note that we have a total of 16 branches in our 8 loop iterations-one of each type per iteration. The BEQ, like all branches at the end of a loop, is taken in all iterations except the last. The BNE depends on the values in array A . As noted, if the condition $\mathrm{A}[\mathrm{i}]>4$ is true, the branch is not taken; if the condition is false, the branch is taken. We can therefore see that the branch is not taken for iterations $0,2,3$, and $5(\mathrm{~A}[\mathrm{i}]=10,6,7$, and 5 , respectively) and taken for iterations $1,4,6$, and 7 $(\mathrm{A}[\mathrm{i}]=4,1,2$, and 3 , respectively). We can now fill out a table like the ones we completed in class to help solve this problem:

| Iteration | Branch | BHT <br> Entry \# | BHT <br> Entry <br> Value | Prediction | Actual Outcome | New BHT Entry Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | BNE | 3 | 00 | NT | NT | 00 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 1 | BNE | 3 | 00 | NT | T | 01 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 2 | BNE | 3 | 01 | NT | NT | 00 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 3 | BNE | 3 | 00 | NT | NT | 00 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 4 | BNE | 3 | 00 | NT | T | 01 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 5 | BNE | 3 | 01 | NT | NT | 00 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 6 | BNE | 3 | 00 | NT | T | 01 |
|  | BEQ | 6 | 11 | T | T | 11 |
| 7 | BNE | 3 | 01 | NT | T | 11 |
|  | BEQ | 6 | 11 | T | NT | 10 |

Of the 16 branches, 5 are mispredicted, so the misprediction rate is $\mathbf{5 1 6}=\mathbf{3 1 . 2 5 \%}$.
2. Correlating branch predictors (50 points) Now assume you have a 4-line, (2,2) correlating branch predictor, with all entries initially set to 00. Assume the initial global history is 00 . Determine the overall accuracy (percentage of correct predictions) of this predictor using the same code as in Problem 1.

Solution: We start this problem in the same manner as the previous one: figure out which lines to access. Note that this table only has $4=2^{2}$ lines, so we need two bits:

BNE: address $44=0 \times 2 \mathrm{C}=0010 \underline{\mathbf{1 1}} 00 \rightarrow$ entry 3
BEQ: address $120=0 \times 78=0111 \underline{10} 00 \rightarrow$ entry 2
The actual branch outcomes remain the same; all that is left is to determine the accuracy of the predictions through a table similar to the one above, only with columns to track the global history:

| Iter. | Global <br> History | Branch | BHT <br> Entry <br> $\#$ | BHT <br> Entry <br> Value | Pred. | Actual <br> Outcome | New BHT <br> Entry <br> Value | New <br> Global <br> Hist |
| :---: | :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 00 | BNE | $[3,0]$ | 00 | NT | NT | 00 | 00 |
|  | 00 | BEQ | $[2,0]$ | 00 | NT | T | 01 | 01 |
| 1 | 01 | BNE | $[3,1]$ | 00 | NT | T | 01 | 11 |
|  | 11 | BEQ | $[2,3]$ | 00 | NT | T | 01 | 11 |
| 2 | 11 | BNE | $[3,3]$ | 00 | NT | NT | 00 | 10 |
|  | 10 | BEQ | $[2,2]$ | 00 | NT | T | 01 | 01 |
| 3 | 01 | BNE | $[3,1]$ | 01 | NT | NT | 00 | 10 |
|  | 10 | BEQ | $[2,2]$ | 01 | NT | T | 11 | 01 |
| 4 | 01 | BNE | $[3,1]$ | 00 | NT | T | 01 | 11 |
|  | 11 | BEQ | $[2,3]$ | 01 | NT | T | 11 | 11 |
| 5 | 11 | BNE | $[3,3]$ | 00 | NT | NT | 00 | 10 |
|  | 10 | BEQ | $[2,2]$ | 11 | T | T | 11 | 01 |
| 6 | 01 | BNE | $[3,1]$ | 01 | NT | T | 11 | 11 |
|  | 11 | BEQ | $[2,3]$ | 11 | T | T | 11 | 11 |
| 7 | 11 | BNE | $[3,3]$ | 00 | NT | T | 01 | 11 |
|  | 11 | BEQ | $[2,3]$ | 11 | T | NT | 10 | 10 |

Of the 16 branches, 10 are mispredicted; the accuracy is the percentage of correct branches: $6 / 16$, or $37.5 \%$.

